Washington Township School District
The mission of the Washington Township Public Schools is to provide a safe, positive, and progressive educational environment that provides opportunity for all students to attain the knowledge and skills specified in the

NJ Learning Standards at all grade levels, so as to ensure their full participation in an ever-changing world as responsible, self-directed and civic-minded citizens.

## Course Title: Honors Calculus

Grade Level(s):
12

| Duration: | Full Year: | X | Semester: | Marking Period: |
| :--- | :--- | :--- | :--- | :--- |
| Course Description: | Honors Calculus is a weighted 5 credit course that follows successful completion of <br> CP Pre-Calculus. Honors Calculus includes topics related to the following: pre- <br> calculus review, limits, techniques of differentiation, applications of differentiation, <br> techniques of integration, and applications of integration. Additionally, topics include <br> problems related to algebraic, trigonometric, exponential, logarithmic, and a strong <br> emphasis placed on working with graphing calculators. Although not required, <br> students should have their own graphing calculator. |  |  |  |
| Grading Procedures: | Each semester will be a composite of quiz scores, test scores, homework, and <br> participation reflecting a student's mastery of the areas outlined above. The student <br> can pass the course with an overall average of 70\%. The individual teacher will <br> explain the grading system to the student. |  |  |  |
| Primary Resources:NJDOE HS Algebra Model Curriculum <br> NJ Student Learning Standards Mathematics (NJSLS-M) <br> CALCULUS with CalcChat and CalcView 2 ${ }^{\text {nd }}$ edition, Ron Larson, and Paul <br> Battaglia; Cengage Learning, LLC. |  |  |  |  |

## Washington Township Principles for Effective Teaching and Learning

- Implementing a standards-based curriculum
- Facilitating a learner-centered environment
- Using academic target language and providing comprehensible instruction
- Adapting and using age-appropriate authentic materials
- Providing performance-based assessment experiences
- Infusing 21st century skills for College and Career Readiness in a global society


## Designed by:

Natalie Barretta

Under the Direction of:
Dr. Carole English

Written:
07/30/2022
Revised:
BOE Approval:

## Unit Title: P - Preparation for Calculus

## Unit Description:

In this unit students will prepare for learning the course calculus. Topics that will be reviewed are graphs and models, linear models, functions and their inverses, and exponential and logarithmic functions. The unit will look at each of their graphs and will fit data to one of these models.

Unit Duration: 2 weeks

## Desired Results

Standard(s):
F.BF.A Build a function that models a relationship between two quantities.
F.BF.B Build new functions from existing functions.
A.SSE.A Interpret the structure of expressions
A.SSE.B Write expressions in equivalent forms to solve problems.
A.APR.A Perform arithmetic operations on polynomials.
A.APR.B Understand the relationship between zeros and factors of polynomials.
A.APR.D Rewrite rational expressions.
F.IF.C Analyze functions using different representations.
F.LE. A Construct and compare linear and exponential models and solve problems.

## Indicators:

F.BF.A. 1 Determine an explicit expression, a recursive process, or steps for calculation from a context.
F.BF.B. 3 Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
A.SSE.A. 1 Interpret expressions that represent a quantity in terms of its context.

- a. Interpret parts of an expression, such as terms, factors, and coefficients.
- b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)$ n as the product of $P$ and a factor not depending on $P 2$. Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)$.


## A.SSE.B.3-4

- 3.Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems
A.APR.A. 1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.B. 3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.D. 6 Rewrite simple rational expressions in different forms; write $a(x) / b(x)$ in the form $q(x)+r(x) / b(x)$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
F.IF.C.8-9
- 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
F.LE.A.1-3
- 1.Distinguish between situations that can be modeled with linear functions and with exponential functions.
- 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- 3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.


## Understandings:

Students will understand that...

- Students can sketch the graph of an equation and find the intercepts of a graph.
- Students can test a graph of symmetry with respect to an axis and the origin.
- The points of intersection of two graphs are the solution to a problem.
- Interpreting mathematical models for real life data helps write equations.
- Students can find the slope of a line passing through two points
- Writing the equations of a line with a given point and slope and interpret slope as a ratio or as a rate in a real-life application brings life to a problem.
- Sketching the graph of a linear equation in slope-intercept form requires the slope and intercept.
- To write equations of lines that are parallel or perpendicular to a given line requires key characteristics of the function.
- Students can use function notation to represent and evaluate a function.
- The domain and range of a function are related to the $x$ and $y$ axis.
- There are different ways to identify different types of transformations of functions.
- Students can classify functions and recognize combinations of functions, fit a linear model to a real-life data set, fit a quadratic model to real-life data, and fit a trigonometric model to a real-life data set. Verify that one function is the inverse of another function.
- Functions have inverses.
- There are properties of trigonometric functions.
- Students need to develop and use properties of exponential functions.
- There is a definition of a number and understand the definition of the natural logarithmic function and develop and use properties of the natural logarithmic function.


## Essential Questions:

- How do you sketch the graph of an equation?
- How can you find the intercepts of a graph?
- How do you test a graph of symmetry with respect to an axis and the origin?
- How do you find the points of intersection of two graphs?
- How can you interpret mathematical models for real-life data?
- How do you find the slope of a line passing through two points?
- How do you write the equations of a line with a given point and slope?
- How do you interpret slope as a ratio or as a rate in a real-life application?
- How do you sketch the graph of a linear equation in slope-intercept form?
- How do you write equations of lines that are parallel or perpendicular to a given line?
- How do you use function notation to represent and evaluate a function?
- How do you find the domain and range of a function?
- How do you sketch the graph of a function?
- How do you identify different types of transformations of functions?
- How do you classify functions and recognize combinations of functions?
- How can you fit a linear model to a real-life data set?
- How can you fit a quadratic model to real-life data?
- How can you fit a trigonometric model to a real-life data set?
- How do you verify that one function is the inverse of another function?
- How do you determine whether a function has an inverse function?
- How do you develop properties of the six trigonometric functions?
- How do you develop and use properties of exponential functions?
- How do you understand the definition of the number e?
- How do you understand the definition of the natural logarithmic function, and develop and use properties of the natural logarithmic function?


## Performance Tasks:

Checkpoints
Homework
WebAssign Assignments
Lesson Quizzes
Mid-Chapter Quiz
Exit / Admit Tickets

## Other Evidence:

WebAssign Remediation
Skill Refresher
Review and Refresh exercises
Vocabulary concept check
Study Strategies
Calculus Workbook Help

## Benchmarks:

Chapter P Test
Summer Assignment Quiz

## Learning Plan

Learning Activities:
*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
Guided notes
Ed Puzzle Notes - optional
Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets
Homework (online WebAssign)

## Unit Content :

## P. 1 Graphs and Models

- Sketch the graph of an equation.
- Find the intercepts of a graph.
- Test a graph for symmetry with respect to an axis and the origin.
- Find the points of intersection of two graphs.
- Fit a mathematical model to a real-life data set.


## P. 2 Linear Models and Rates of Change

- Find the slope of a line passing through two points.
- Write the equation of a line with a given point and slope.
- Interpret slope as a ratio or as a rate in a real-life application.
- Sketch the graph of a linear equation in slope-intercept form.
- Write equations of lines that are parallel or perpendicular to a given line.


## P. 3 Functions and Their Graphs

- Use function notation to represent and evaluate a function.
- Find the domain and range of a function.
- Sketch the graph of a function.
- Identify different types of transformations of functions.
- Classify functions and recognize combinations of functions.


## P. 4 Inverse Functions

- Verify that one function is the inverse function of another functions.
- Determine whether the function has an inverse function.
- Develop properties of the six inverse trigonometric functions.


## P. 5 Exponential and Logarithmic Functions

- Develop and use the properties of exponential functions.
- Understand the definition of e.
- Understand the definition of the natural logarithmic function and develop and use properties of the natural logarithmic function.


## Chapter Review

## Chapter Test

## Resources:

WebAssign online textbook and practice
QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.
Cengage.com for video solutions of selected exercises.

| Advanced Learners | - Invite students to explore different points of view on a topic of study and compare the two. <br> - Assign a leadership role in classroom learning <br> - Determine where student's interests lie and capitalize on their inquisitiveness. <br> - Expose students to a selection and use of specialized resources |
| :---: | :---: |
| Struggling Learners | - Be flexible with time frames and deadlines <br> - Create planned opportunities for interaction between individuals in the classroom: cooperative and collaborative learning, pair and share with peers <br> - Group students <br> - Intentional scheduling/grouping with student/teacher of alternative background <br> - Provide support as at-risk students move through all levels of knowledge acquisition <br> - Tap prior knowledge |
| English Language Learners | - Accommodate with completed study guides to assist with preparation on tests <br> - Allow students to give responses in a form (oral or written) that's easier for him/her <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience <br> - Establish a framework allowing ELL students to understand and assimilate new ideas and information <br> - Focus on domain specific vocabulary and keywords <br> - Give alternate or paper copies to accommodate electronic assignments. <br> - Have another student share class notes with the ELL student. <br> - Intentional scheduling/grouping with student/teacher of language if possible <br> - Mark texts with a highlighter. <br> - Take more time to complete a task, project, or test. <br> - Use manipulatives, graphic organizer, and real objects when possible <br> - Use visual presentations/verbal materials (ex: word webs and visual organizers). |
| Special Needs Learners | - Accommodate with completed study guides to assist with preparation on tests. <br> - Allow more time to complete task, project, or test <br> - Allow students to give responses in a form ( oral or written) that's easier for him <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Give alternate or paper copies to replace electronic assignments <br> - Have another student share class notes with the special needs learner. <br> - Higher level reasoning and questioning would have less weight than other assignments. <br> - Receive study skill instructions. <br> - Work with fewer items per page or line and/or materials in a larger print. |
| Learners with a 504 | Refer to page four in the Parent and Educator Resource Guide to Section 504 to assist in the development of appropriate plans. |

## Interdisciplinary Connections

## Indicators:

## ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

## Computer Science and Design Thinking

8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

## Technology Education

9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.

## Life Literacies \& Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

## Integration of 21 ${ }^{\text {st }}$ Century Skills

Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and critical thinking.

## Unit Title: 1 Limits and Their Properties <br> Unit Description:

The limit process is a fundamental concept of calculus that separates it from algebra and trigonometry. In this unit, students will show how to define limits and show how to evaluate limits analytically, graphically, and numerically. Students will also show how to define continuity and determine if a function is continuous.

## Unit Duration: 7 weeks

## Desired Results

## Standard(s):

N.Q.A Reason quantitatively and use units to solve problems.
F.BF.B Build new functions from existing functions.
A.SSE.A Interpret the structure of expressions.
A.APR.A Perform arithmetic operations on polynomials.
A.APR.B Understand the relationship between zeros and factors of polynomials.
A.APR.D Rewrite rational expressions.
F.IF.C Analyze functions using different representations.

## Indicators:

N.Q.A. 1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
F.BF.B. 3 Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
A.SSE.A. 1 Interpret expressions that represent a quantity in terms of its context.

- 1a. Interpret parts of an expression, such as terms, factors, and coefficients.
- 1b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r) n$ as the product of $P$ and a factor not depending on $P 2$. Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)$.
A.APR.A. 1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. A.APR.B. 3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.D. 6 Rewrite simple rational expressions in different forms; write $a(x) / b(x)$ in the form $q(x)+r(x) / b(x)$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
F.IF.C.8-9
- 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.


## Understandings:

Students will understand that...

- Calculus is the mathematics of change.
- The tangent line problem is basic to calculus.
- The area problem is also basic to calculus.
- Students can estimate a limit using a numerical or graphical approach.
- There are different ways that a limit can fail to exist
- They can use a formal definition of limit to evaluate a limit using properties.
- There are strategies that can be developed to help find limits
- Students need to evaluate a limit using the dividing out technique and rationalizing technique when the basic properties fail.
- Students can evaluate a limit using the squeeze theorem.
- To determine continuity at a point and continuity on an open interval students can apply the properties of continuity.
- To determine one-sided limits and continuity on a closed interval the function must be evaluated.
- Students can use properties of continuity.
- Students can use the Intermediate Value Theorem to help determine the behavior of a function on a continuous closed interval.
- Students can determine infinite limits from the left and from the right.
- Students can find and sketch the vertical asymptotes of the graph of a function.


## Essential Questions:

- What calculus is and how it compares with precalculus?
- What is the tangent line problem?
- What is the area problem?
- How do you estimate a limit using a numerical or graphical approach?
- How can a limit fail to exist?
- What is the formal definition of limit?
- How do you evaluate a limit using properties of limits?
- How can you develop and use a strategy for finding limits?
- How do you evaluate a limit using the dividing out technique?
- How do you evaluate a limit using the rationalizing technique?
- How do you evaluate a limit using the squeeze theorem?
- How do you determine continuity at a point and continuity on an open interval?
- How do you determine one-sided limits and continuity on a closed interval?
- How do you use properties of continuity?
- How do you use the Intermediate Value Theorem?
- How do you determine infinite limits from the left and from the right?
- How do you find and sketch the vertical asymptotes of the graph of a function?

Assessment Evidence
Performance Tasks:

Checkpoints
Homework
WebAssign Assignments
Lesson Quizzes
Mid-Chapter Quiz
Exit / Admit Tickets

## Other Evidence:

WebAssign Remediation
Skill Refresher
Review and Refresh exercises
Vocabulary concept check
Study Strategies
Calculus Workbook Help

## Benchmarks:

Chapter 1 Test
Chapter 1 Project - Graphs and Limits of Trigonometric Functions

## Learning Activities:

*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
Guided notes
Ed Puzzle Notes - optional
Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets
Homework (online Math XL)

## Unit Content :

### 1.1 A Preview of Calculus

- Understand what calculus is and how it compares with precalculus.
- Understand that the tangent line problem is basic to calculus.
- Understand that the area problem is also basic to calculus.


### 1.2 Finding Limits Graphically and Numerically

- Estimate a limit using a numerical and graphical approach.
- Learn different ways that a limit can fail to exist.
- Study and use a formal definition of limit.


### 1.3 Evaluating Limits Analytically

- Evaluate a limit using properties of limits.
- Develop and use a strategy for finding limits.
- Evaluate a limit using the dividing out technique.
- Evaluate a limit using the rationalizing technique.
- Evaluate a limit using the Squeeze Theorem.


### 1.4 Continuity and One-Sided Limits

- Determine continuity at a point and continuity on an open interval.
- Determine one-sided limits and continuity on a closed interval.
- Use properties of continuity.
- Understand and use the Intermediate Value Theorem.


### 1.5 Infinite Limits

- Determine infinite limits from the left and from the right.
- Find and sketch the vertical asymptotes of the graph of a function.


### 1.6 Limits at Infinity

- Determine (finite) limits at infinity.
- Determine the horizontal asymptotes, if any, of the graph of a function.
- Determine the infinite limits at infinity.


## Chapter Review

## Chapter Test

Resources:

WebAssign online textbook and practice
QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises. Cengage.com for video solutions of selected exercises.

| Advanced Learners | Invite students to explore different points of view on a topic of study and compare the two. <br> - Assign a leadership role in classroom learning <br> - Determine where student's interests lie and capitalize on their inquisitiveness <br> - Expose students to a selection and use of specialized resources |
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| English Language Learners | - Accommodate with completed study guides to assist with preparation on tests <br> - Allow students to give responses in a form (oral or written) that's easier for him/her <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience <br> - Establish a framework allowing ELL students to understand and assimilate new ideas and information <br> - Focus on domain specific vocabulary and keywords <br> - Give alternate or paper copies to accommodate electronic assignments. <br> - Have another student share class notes with the ELL student. <br> - Intentional scheduling/grouping with student/teacher of language if possible <br> - Mark texts with a highlighter. <br> - Take more time to complete a task, project, or test. <br> - Use manipulatives, graphic organizer, and real objects when possible <br> - Use visual presentations/verbal materials (ex: word webs and visual organizers). |
| Special Needs Learners | - Accommodate with completed study guides to assist with preparation on tests. <br> - Allow more time to complete task, project, or test <br> - Allow students to give responses in a form ( oral or written) that's easier for him <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Give alternate or paper copies to replace electronic assignments <br> - Have another student share class notes with the special needs learner. <br> - Higher level reasoning and questioning would have less weight than other assignments. <br> - Receive study skill instructions. <br> - Work with fewer items per page or line and/or materials in a larger print. |
| Learners with a 504 | Refer to page four in the Parent and Educator Resource Guide to Section $\underline{504}$ to assist in the development of appropriate plans. |

## Interdisciplinary Connections

## Indicators:

## ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

## Computer Science and Design Thinking

8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

## Technology Education

9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems,

## Life Literacies \& Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

## Integration of 21 ${ }^{\text {st }}$ Century Skills

Indicators:
From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21 st century learning in this unit include collaboration, critical thinking, and communication.

| Unit Title: 2 Differentiation |
| :--- |
| Unit Description: |
| In this unit, students will explore derivatives as the slope of the tangent to a curve at a particular point. |
| Students will begin with the limit definition of the derivative, and then continue with the basic derivative rules, |
| the product rule, the quotient rule, the chain rule, and implicit differentiation. Students also explore applications |
| such as the tangent line problem and rates of change. |
| Unit Duration: 8 weeks |

## Desired Results

Standard(s):
F.BF.A Build a function that models a relationship between two quantities.
F.BF.B Build new functions from existing functions.
A.SSE.A Interpret the structure of expressions.
A.SSE.B Write expressions in equivalent forms to solve problems.
A.APR.A Perform arithmetic operations on polynomials.
A.APR.B Understand the relationship between zeros and factors of polynomials.
A.APR.D Rewrite rational expressions.
F.IF.C Analyze functions using different representations.

## Indicators:

F.BF.A. 1 Determine an explicit expression, a recursive process, or steps for calculation from a context.
F.BF.B. 3 Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
A.SSE.A. 1 Interpret expressions that represent a quantity in terms of its context.

- 1a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r) n$ as the product of $P$ and a factor not depending on $P$
- 2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-$ $\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)$.
A.SSE.B.3-4 3.Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1 ) and use the formula to solve problems.
A.APR.A. 1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. A.APR.B. 3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.D. 6 Rewrite simple rational expressions in different forms; write $a(x) / b(x)$ in the form $q(x)+r(x) / b(x)$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.


## F.IF.C.8-9

- 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.


## Understandings:

Students will understand that...

- Students can find the slope of tangent line to a curve at a point.
- Students can use the limit to find the derivative of a function
- There is a difference between differentiability and continuity.
- Students can find the derivative of a function using the constant rule, power rule, and the constant multiple rule.
- Students can find the derivative of a function using the sum and difference rules.
- There are derivatives of sine function and cosine functions and exponential functions.
- Derivatives are used to find the rate of change.
- The power rule and quotient rule are used to find derivatives.
- There are derivatives of trigonometric functions.
- To find the derivative of a composite function the chain rule can be used.
- To find the derivative of a function can be determined using the general power rule and/or algebra.
- The chain rule can be used to find the derivative of a transcendental function.
- There are derivatives of a function involving the natural logarithmic function.
- Exponential functions that have bases other than e are also differentiable.
- Students can distinguish between functions written in implicit form and explicit form.
- Use implicit differentiation to find the derivative of a function
- Find derivatives of functions using logarithmic differentiation.
- There is a derivative of an inverse function.
- How to differentiate an inverse trigonometric function.
- Students can find a related rate and use related rates to solve real-life problems.
- Newton's Method is a way to approximate a zero of a function.


## Essential Questions:

- How do you find the slope of tangent line to a curve at a point?
- How do you use the limit to find the derivative of a function?
- What is the relationship between differentiability and continuity?
- How do you find the derivative of a function using the constant rule, power rule, and constant multiple rule?
- How do you find the derivative of a function using the sum and difference rules?
- How do you find the derivative of sine function and cosine functions?
- How do you find the derivatives of exponential functions?
- How do you use derivatives to find the rate of change?
- How do you find the derivative of a function using the power rule and quotient rule?
- How do you find the derivative of a trigonometric function and higher-order functions?
- How do you find the derivative of a composite function using the chain rule?
- How do you find the derivative of a function using the general power rule and algebra?
- How do you find the derivative of a transcendental function using the chain rule?
- How do you find the derivative of a function involving the natural logarithmic function?
- How do you define and differentiate exponential functions that have bases other than e?
- How do you distinguish between functions written in implicit form and explicit form?
- How do you us e implicit differentiation to find the derivative of a function?
- How do you find derivatives of functions using logarithmic differentiation?
- How do you find the derivative of an inverse function?
- How do you differentiate an inverse trigonometric function?
- How do you find a related rate and how do you use them to solve real life problems?
- How do you approximate a zero of a function using Newton's method?

Assessment Evidence

Performance Tasks:
Checkpoints
Homework
WebAssign Assignments

## Other Evidence:

WebAssign Remediation
Skill Refresher
Review and Refresh exercises

| Lesson Quizzes | Vocabulary concept check |
| :--- | :--- |
| Mid-Chapter Quiz | Study Strategies |
| Exit / Admit Tickets | Calculus Workbook Help |

## Benchmarks:

Chapter 2 Assessment
Chapter 2 Project - Optical Illusions

## Learning Plan

## Learning Activities:

*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
Guided notes
Ed Puzzle Notes - optional
Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets
Homework (online Math XL)

## Unit Content:

2.1 The Derivative and The Tangent Line Problem

- Find the slope of tangent line to a curve at a point.
- Use the limit to find the derivative of a function.
- Understanding the relationship between differentiability and continuity.
- Find the derivative of a function given by a table or a graph.


### 2.2 Basic Differentiation Rules and Rates of Change

- Find the derivative of a function using the constant rule.
- Find the derivative of a function using the power rule.
- Find the derivative of a function using the constant multiple rule.
- Find the derivative of a function using the sum and difference rules.
- Find the derivative of sine function and cosine functions.
- Find the derivatives of exponential functions.
- Use derivatives to find the rate of change.


### 2.3 Product and Quotient Rules and Higher-Order Derivatives

- Find the derivative of a function using the Product Rule.
- Find the derivative of a function using the Quotient Rule.
- Find the derivative of a trigonometric function.
- Find a higher-order derivative of a function.


### 2.4 The Chain Rule

- Find the derivative of a composite function using the Chain Rule.
- Find the derivative of a function using the General Power Rule.
- Find the derivative of a function using algebra.
- Find the derivative of a transcendental function using the Chain Rule.
- Find the derivative of a function involving the natural logarithmic function.
- Define and differentiate exponential functions that have bases other than e


### 2.5 Implicit Differentiation

- Distinguish between functions written in implicit form and explicit form.
- Use implicit differentiation to find the derivative of a function.
- Find derivatives of functions using logarithmic differentiation.


### 2.6 Derivatives of Inverse Functions

- Find the derivative of an inverse function.
- Differentiate an inverse trigonometric function


### 2.7 Related Rates

- Find a related rate.
- Use related rates to solve real-life problems.


### 2.8 Newton's Method

- Approximate a zero of a function using Newton's method.


## Chapter Review

## Chapter Test

## Resources:

WebAssign online textbook and practice
QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.
Cengage.com for video solutions of selected exercises.

| Advanced Learners | Invite students to explore different points of view on a topic of study and compare the two. <br> - Assign a leadership role in classroom learning <br> - Determine where student's interests lie and capitalize on their inquisitiveness <br> - Expose students to a selection and use of specialized resources |
| :---: | :---: |
| Struggling Learners | - Be flexible with time frames and deadlines <br> - Create planned opportunities for interaction between individuals in the classroom: cooperative and collaborative learning, pair and share with peers <br> - Group students <br> - Intentional scheduling/grouping with student/teacher of alternative background <br> - Provide support as at-risk students move through all levels of knowledge acquisition <br> - Tap prior knowledge |
| English Language Learners | - Accommodate with completed study guides to assist with preparation on tests <br> - Allow students to give responses in a form (oral or written) that's easier for him/her <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience <br> - Establish a framework allowing ELL students to understand and assimilate new ideas and information <br> - Focus on domain specific vocabulary and keywords <br> - Give alternate or paper copies to accommodate electronic assignments. <br> - Have another student share class notes with the ELL student. <br> - Intentional scheduling/grouping with student/teacher of language if possible <br> - Mark texts with a highlighter. <br> - Take more time to complete a task, project, or test. <br> - Use manipulatives, graphic organizer, and real objects when possible <br> - Use visual presentations/verbal materials (ex: word webs and visual organizers). |
| Special Needs Learners | - Accommodate with completed study guides to assist with preparation on tests. <br> - Allow more time to complete task, project, or test <br> - Allow students to give responses in a form ( oral or written) that's easier for him <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Give alternate or paper copies to replace electronic assignments <br> - Have another student share class notes with the special needs learner. <br> - Higher level reasoning and questioning would have less weight than other assignments. <br> - Receive study skill instructions. <br> - Work with fewer items per page or line and/or materials in a larger print. |
| Learners with a 504 | Refer to page four in the Parent and Educator Resource Guide to Section $\underline{504}$ to assist in the development of appropriate plans. |

## Interdisciplinary Connections

## Indicators:

## ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

## Computer Science and Design Thinking

8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

## Technology Education

9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.

## Life Literacies \& Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

## Integration of $21^{\text {st }}$ Century Skills

## Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and communication.

| Unit Title: 3 Applications of Differentiation |
| :--- |
| Unit Description: |
| In this unit, students will use calculus to analyze graphs of functions. Students will find the minimum and |
| maximum points on the graph as well as where the graph is concave up or concave down. Students will use |
| these techniques to sketch the graph of a function. Students will also use calculus to solve optimization |
| problems. |

## Unit Duration: 8 weeks

## Desired Results

Standard(s):
N.Q.A Reason quantitatively and use units to solve problems.
G.C.A Understand and apply theorems about circles.
F.BF.A Build a function that models a relationship between two quantities
A.SSE.A Interpret expressions that represent a quantity in terms of its context.
A.SSE.B Write expressions in equivalent forms to solve problems
A.CED.A Create equations that describe numbers or relationships
A.REI.B Solve equations and inequalities in one variable
A.APR.A Perform arithmetic operations on polynomials
A.APR.B Understand the relationship between zeros and factors of polynomials
A.APR.D Rewrite rational expressions
F.IF.B Interpret functions that arise in applications in terms of the context
F.LE. A Construct and compare linear and exponential models and solve problems
F.LE.B Interpret expressions for functions in terms of the situation they model

## Indicators:

N.Q.A. 1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
G.C.A. 2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
F.BF.A. 1 Determine an explicit expression, a recursive process, or steps for calculation from a context.
A.SSE.A. 1 Interpret expressions that represent a quantity in terms of its context. 1 a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r) n$ as the product of $P$ and a factor not depending on P 2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)$.
A.SSE.B.3-4 3.Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1 ), and use the formula to solve problems
A.CED.A.1-4

- 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- 2 Create equations in two or more variables to represent relationships between quantities, graph equations on coordinate axes with labels and scales.
- 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
- 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V=I R$ to highlight resistance $R$.
A.REI.B.3-4a
- 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- 4. Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x-p) 2=q$ that has the same solutions. Derive the quadratic formula from this form.
A.APR.A. 1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.B. 3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.D. 6 Rewrite simple rational expressions in different forms; write $a(x) / b(x)$ in the form $q(x)+r(x) / b(x)$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
F.IF.B 4-6
- 4.For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.
- 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
F.LE.A.1-3
- 1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
- 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- 3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
F.LE.B. 5 Interpret the parameters in a linear or exponential function in terms of a context.


## Understandings:

Students will understand that...

- The definition of extrema of a function on an interval.
- There is definition of relative extrema of a function on an open interval.
- Students can find extrema on a closed interval.
- The Rolle's Theorem guarantees the existence of an extreme value in the interior on a closed interval
- The Mean Value Theorem guarantees the existence of a tangent line that is parallel to a secant line through given points and that there is a point in an open interval $(a, b)$ which the instantaneous rate of change is equal to the average rate of change in the closed interval.
- Students can find intervals on which a function is increasing or decreasing.
- The First Derivative Test can be used to find relative extrema of a function.
- How to determine intervals on which a function is concave upward and concave downward.


## Essential Questions:

- What is the definition of extrema of a function on an interval?
- What is the definition of relative extrema of a function on an open interval?
- How do you find extrema on a closed interval?
- How do you use Rolle's theorem?
- How do you use the Mean value theorem?
- How do you determine intervals on which a function is increasing or decreasing?
- How do you apply the First derivative test and find relative extrema of a function?
- How do you determine intervals on which a function is concave upward and concave downward?
- How do you find any points of inflection of the graph of a function?
- How do you apply the Second derivative test to find relative extrema of a function?
- How do you determine (finite) limits at infinity?
- How do you determine the horizontal asymptotes, if any, of the graph of a function?
- Students can find any points of inflection of the graph of a function.
- How to apply the Second Derivative Test to find relative extrema of a function.
- There are (finite) limits at infinity.
- How to find the horizontal asymptotes, if any, of the graph of a function and determine infinite limits at infinity.
- There are strategies to analyze and sketch the graph of a function.
- Students can solve applied minimum and maximum problems to find needed information.
- The concept of a tangent line approximation is a basic in calculus.
- Students can compare the value of the differential, dy, with the actual change in $y$.
- Students can estimate a propagated error using a differential.
- How to find the differential of a function using differentiation formulas.
- How do you determine infinite limits at infinite?
- How do you analyze and sketch the graph of a function?
- How do you solve applied minimum and maximum problems?
- What is the concept of a tangent line approximation?
- How do you compare the value of the differential, dy, with the actual change in $y$ ?
- How do you estimate a propagated error using a differential?
- How do you find the differential of a function using differentiation formulas?

Assessment Evidence

| Performance Tasks: | Other Evidence: |
| :--- | :--- |
| Checkpoints | WebAssign Remediation |
| Homework | Skill Refresher |
| WebAssign Assignments | Review and Refresh exercises |
| Lesson Quizzes | Vocabulary concept check |
| Mid-Chapter Quiz | Study Strategies |
| Exit / Admit Tickets | Calculus Workbook Help |

## Benchmarks:

Chapter 3 Assessment
Chapter 3 Project - Areas of Triangles and/or Stopping Distance
Sketch Packet

Learning Plan

## Learning Activities:

*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
Guided notes
Ed Puzzle Notes - optional
Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets
Homework (online Math XL)

## Unit Content:

### 3.1 Extrema on an Interval

- Understand the definition of extrema of a function on an interval.
- Understand the definition of relative extrema of a function on an open interval.
- Find extrema on a closed interval.
3.2 Rolle's Theorem and The Mean Value Theorem
- Understand and use Rolle's Theorem.
- Understand and use The Mean Value Theorem.
3.3 Increasing and Decreasing Functions and the First Derivative Test
- Determine intervals on which a function is increasing or decreasing.
- Apply the First Derivative Test and find relative extrema of a function.


### 3.4 Concavity and The Second Derivative Test

- Determine intervals on which a function is concave upward and concave downward.
- Find any points of inflection of the graph of a function.
- Apply the Second Derivative Test to find relative extrema of a function.


### 3.5 A Summary of Curve Sketching

- Analyze and sketch the graph of a function


### 3.6 Optimization Problems

- Solve applied minimum and maximum problems.


### 3.7 Linear Approximations and Differentials

- Understand the concept of a tangent line approximation.
- Compare the value of the differential, dy, with the actual change in $y$.
- Estimate a propagated error using a differential.
- Find the differential of a function using differentiation formulas


## Chapter Review

## Chapter Test

## Resources:

WebAssign online textbook and practice
QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.
Cengage.com for video solutions of selected exercises.

| Advanced Learners | - Invite students to explore different points of view on a topic of study and compare the two. <br> - Assign a leadership role in classroom learning <br> - Determine where student's interests lie and capitalize on their inquisitiveness <br> - Expose students to a selection and use of specialized resources |
| :---: | :---: |
| Struggling Learners | - Be flexible with time frames and deadlines <br> - Create planned opportunities for interaction between individuals in the classroom: cooperative and collaborative learning, pair and share with peers <br> - Group students <br> - Intentional scheduling/grouping with student/teacher of alternative background <br> - Provide support as at-risk students move through all levels of knowledge acquisition <br> - Tap prior knowledge |
| English Language Learners | - Accommodate with completed study guides to assist with preparation on tests <br> - Allow students to give responses in a form (oral or written) that's easier for him/her <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience <br> - Establish a framework allowing ELL students to understand and assimilate new ideas and information <br> - Focus on domain specific vocabulary and keywords <br> - Give alternate or paper copies to accommodate electronic assignments. <br> - Have another student share class notes with the ELL student. <br> - Intentional scheduling/grouping with student/teacher of language if possible <br> - Mark texts with a highlighter. <br> - Take more time to complete a task, project, or test. <br> - Use manipulatives, graphic organizer, and real objects when possible <br> - Use visual presentations/verbal materials (ex: word webs and visual organizers). |
| Special Needs Learners | - Accommodate with completed study guides to assist with preparation on tests. <br> - Allow more time to complete task, project, or test <br> - Allow students to give responses in a form ( oral or written) that's easier for him <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Give alternate or paper copies to replace electronic assignments <br> - Have another student share class notes with the special needs learner. <br> - Higher level reasoning and questioning would have less weight than other assignments. <br> - Receive study skill instructions. <br> - Work with fewer items per page or line and/or materials in a larger print. |
| Learners with a 504 | Refer to page four in the Parent and Educator Resource Guide to Section $\underline{504}$ to assist in the development of appropriate plans. |

## Indicators:

## ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

## Computer Science and Design Thinking

8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

## Technology Education

9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.

## Life Literacies \& Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

## Integration of $21^{\text {st }}$ Century Skills

## Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and critical thinking.

| Unit Title: 4 Integration |
| :--- |
| Unit Description: |
| In this unit, students will examine how instantaneous changes accumulate over an interval to produce a |
| function. Students will find that this accumulation results in the area under a curve, or the value of an integral |
| over a specific interval. Students will also see that the Fundamental Theorem of Calculus brings differentiation |
| and integration together. |

## Desired Results

Standard(s):
F.BF.A Build a function that models a relationship between two quantities.
F.BF.B Build new functions from existing functions.
A.SSE.A Interpret the structure of expressions.
A.SSE.B Write expressions in equivalent forms to solve problems.
A.APR.A Perform arithmetic operations on polynomials.
A.APR.B Understand the relationship between zeros and factors of polynomials.
A.APR.D Rewrite rational expressions.
F.IF.C Analyze functions using different representations.

## Indicators:

F.BF.A. 1 Determine an explicit expression, a recursive process, or steps for calculation from a context.
F.BF.B. 3 Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
A.SSE.A. 1 Interpret expressions that represent a quantity in terms of its context.

- 1a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r) n$ as the product of $P$ and a factor not depending on $P$
- 2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-$ $\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)$.
A.SSE.B.3-4 3.Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1 ) and use the formula to solve problems.
A.APR.A. 1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.B. 3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.D. 6 Rewrite simple rational expressions in different forms; write $a(x) / b(x)$ in the form $q(x)+r(x) / b(x)$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
F.IF.C.8-9
- 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.


## Understandings:

Students will understand that...

- Students can write the general solution of a differential equation and use indefinite integral notation for antiderivatives.
- Students can use basic integration rules to find antiderivatives.
- How to find a particular solution of a differential equation.
- Students can use sigma notation to write and evaluate a sum.
- Students can approximate the area of a plane region and find the area of a plane region using limits.
- The definition of a Riemann Sum approximates an integral of a finite sum.
- Students can evaluate a definite integral using limits and evaluate a definite integral using properties of definite integrals.
- The Fundamental Theorem of Calculus is used to evaluate a definite integral.
- The Mean Value Theorem of Integrals is used when a function is continuous and takes on the average value of a function over a closed interval.
- The Second Fundamental Theorem of Calculus tells us when a function is continuous, it has an antiderivative.
- The Net Change Theorem gives the total change of a function if the derivative is the rate of a change of quantity.
- How to use pattern recognition to find an indefinite integral.
- How to use a change of variables to find an indefinite integral and how to use the General Power Rule for Integration to find an indefinite integral.
- Students can use a change of variables to evaluate a definite integral.
- To evaluate a definite integral involving an even or odd function students must interpret the function.
- Students can approximate a definite integral using the Trapezoidal Rule and the Simpson's Rule.
- Students can use the Log Rule for Integration to integrate a rational function and integrate trigonometric functions.
- Students can integrate functions whose antiderivatives involve inverse trigonometric functions.
- Students can use the method of completing the square to integrate a function.


## Essential Questions:

- How do you write the general solution of a differential equation and use indefinite integral notation for antiderivatives?
- How do you use basic integration rules to find antiderivatives?
- How can you find a particular solution of a differential equation?
- How do you use sigma notation to write and evaluate a sum?
- What is the concept of area?
- How do you approximate the area of a plane region?
- How do you find the area of a plane region using limits?
- What is the definition of a Riemann sum?
- How do you evaluate a definite integral using limits?
- How do you evaluate a definite integral using properties of definite integrals?
- How do you evaluate a definite integral using the Fundamental Theorem of Calculus?
- How do you use the Mean Value Theorem of Integrals?
- How do you find the average value of a function over a closed interval?
- How do you use the Second Fundamental Theorem of Calculus?
- How do you use the Net Change Theorem?
- How do you use pattern recognition to find an indefinite integral?
- How do you use a change of variables to find an indefinite integral?
- How do you use the General Power Rule for Integration to find an indefinite integral?
- How do you use a change of variables to evaluate a definite integral?
- How do you evaluate a definite integral involving an even or odd function?
- How do you approximate a definite integral using the Trapezoidal Rule?
- How do you approximate a definite integral using the Simpson's Rule?
- How do you use the Log Rule for Integration to integrate a rational function?
- How do you integrate trigonometric functions?
- How do you integrate functions whose antiderivatives involve inverse trigonometric functions?
- How do you use the method of completing the square to integrate a function?


## Performance Tasks:

Checkpoints
Homework
WebAssign Assignments
Lesson Quizzes
Mid-Chapter Quiz
Exit / Admit Tickets

Other Evidence:
WebAssign Remediation
Skill Refresher
Review and Refresh exercises
Vocabulary concept check
Study Strategies
Calculus Workbook Help

## Benchmarks:

Chapter 4 Assessment
Chapter 4 Project - Demonstrating the Fundamental Theorem

## Learning Plan

Learning Activities:
*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
Guided notes
Ed Puzzle Notes - optional
Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets
Homework (online Math XL)

## Unit Content:

### 4.1 Antiderivatives and Indefinite Integration

- Write the general solution of a differential equation and use indefinite integral notation for antiderivatives
- Use basic integration rules to find antiderivatives.
- Find a particular solution of a differential equation.


### 4.2 Area

- Use sigma notation to write and evaluate a sum.
- Understand the concept of area.
- Approximate the area of a plane region.
- Find the area of a plane region using limits


### 4.3 Riemann Sums and Definite Integrals

- Understand the definition of a Riemann sum.
- Evaluate a definite integral using limits.
- Evaluate a definite integral using properties of definite integrals.
- Approximate a definite integral using the Trapezoidal Rule.


### 4.4 The Fundamental Theorem of Calculus

- Evaluate a definite integral using the Fundamental Theorem of Calculus.
- Understand and use the Mean Value Theorem of Integrals.
- Find the average value of a function over a closed interval.
- Understand and use the Second Fundamental Theorem of Calculus.


### 4.5 The Net Change Theorem

- Understand and use the Net Change Theorem.


### 4.6 Integration by Substitution

- Use pattern recognition to find an indefinite integral.
- Use a change of variables to find an indefinite integral.
- Use the General Power Rule for Integration to find an indefinite integral.
- Use a change of variables to evaluate a definite integral.
- Evaluate a definite integral involving an even or odd function.


### 4.7 The Natural Logarithmic Function: Integration

- Use the Log Rule for Integration to integrate a rational function.
- Integrate trigonometric functions.


### 4.8 Inverse Trigonometric Functions: Integration

- Integrate functions whose antiderivatives involve inverse trigonometric functions.
- Use the method of completing the square to integrate a function.
- Review the basic integration rules involving elementary functions.

Chapter Review
Chapter Test

## Resources:

WebAssign online textbook and practice
QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.
Cengage.com for video solutions of selected exercises.

| Advanced Learners | - Invite students to explore different points of view on a topic of study and compare the two. <br> - Assign a leadership role in classroom learning <br> - Determine where student's interests lie and capitalize on their inquisitiveness <br> - Expose students to a selection and use of specialized resources |
| :---: | :---: |
| Struggling Learners | - Be flexible with time frames and deadlines <br> - Create planned opportunities for interaction between individuals in the classroom: cooperative and collaborative learning, pair and share with peers <br> - Group students <br> - Intentional scheduling/grouping with student/teacher of alternative background <br> - Provide support as at-risk students move through all levels of knowledge acquisition <br> - Tap prior knowledge |
| English Language Learners | - Accommodate with completed study guides to assist with preparation on tests <br> - Allow students to give responses in a form (oral or written) that's easier for him/her <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience <br> - Establish a framework allowing ELL students to understand and assimilate new ideas and information <br> - Focus on domain specific vocabulary and keywords <br> - Give alternate or paper copies to accommodate electronic assignments. <br> - Have another student share class notes with the ELL student. <br> - Intentional scheduling/grouping with student/teacher of language if possible <br> - Mark texts with a highlighter. <br> - Take more time to complete a task, project, or test. <br> - Use manipulatives, graphic organizer, and real objects when possible <br> - Use visual presentations/verbal materials (ex: word webs and visual organizers). |
| Special Needs Learners | - Accommodate with completed study guides to assist with preparation on tests. <br> - Allow more time to complete task, project, or test <br> - Allow students to give responses in a form ( oral or written) that's easier for him <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Give alternate or paper copies to replace electronic assignments <br> - Have another student share class notes with the special needs learner. <br> - Higher level reasoning and questioning would have less weight than other assignments. <br> - Receive study skill instructions. <br> - Work with fewer items per page or line and/or materials in a larger print. |
| Learners with a 504 | Refer to page four in the Parent and Educator Resource Guide to Section $\underline{504}$ to assist in the development of appropriate plans. |

## Indicators:

## ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

## Computer Science and Design Thinking

8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

## Technology Education

9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.

## Life Literacies \& Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

## Integration of $21^{\text {st }}$ Century Skills

## Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and communication.

